

Cosmological Magnetic Fields vs. CMB

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ABSTRACT

I present a short review of the effects of a cosmological magnetic field on the CMB temperature and polarization anisotropies. Various possibilities for constraining the magnetic field amplitude are discussed.

Cosmological primordial seed magnetic fields were proposed to explain the existence of observed $\sim 10^{-6}$ G magnetic fields in galaxies and clusters (see, e.g., Widrow 2002, Giovaninni 2003, and references therein). To preserve approximate spatial isotropy a seed vector magnetic field has to be small and hence can be treated as a first order term in perturbation theory. If the energy density parameter of a primordial magnetic field, $\Omega_B = B^2/(4\pi\rho_{cr})$ (where ρ_{cr} is the critical density), is five or six of order of magnitude less than that of the radiation (photons), $\Omega_B \sim 10^{-6} - 10^{-5}\Omega_\gamma$, this is still of the order of the current accuracy of CMB measurements (Bennett et al. 2003), so we might expect that such a field strength ($B \sim 10^{-8} - 10^{-9}$ Gauss) could leave detectable traces on CMB temperature or polarization anisotropies.

Primordial magnetic fields could be generated during early epochs of the Universe, such as during inflation, or the electroweak phase transition, or might be generated by primordial turbulence (for reviews see Grasso and Rubinstein 2001, Widrow 2002, Giovaninni 2003). Cosmological magnetic fields induce scalar (density), vector (vorticity) and tensor (gravitational waves) fluctuations, and through them influence the CMB temperature and polarization anisotropies (see Mack et al. 2002 and references therein). Hence precise CMB measurements (Bennett et al. 2003) can be used to constrain primordial magnetic fields. An interesting possibility is to consider the rotation of the CMB polarization plane due to the Faraday effect (Kosowsky and Loeb 1996).

The simplest illustrative case to consider is a homogeneous magnetic field (Giovaninni and Shaposhnikov 1998), which generates magnetosonic and Alfvén waves. Due to the

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rescaling of sound velocity in a cosmological model with a homogeneous magnetic field: $c_S^2 \rightarrow c_S^2 + v_A^2$ (where $v_A = B/\sqrt{4\pi(\rho + p)}$ is the Alfvén speed), the influence of fast magnetosonic wave propagation on CMB anisotropies consists of shifts in the acoustic peak positions (Adams et al. 1996).

In standard cosmology vector perturbations decay with time and so do not affect the CMB. The presence of a homogeneous magnetic field alters this situation: such a field supports Alfvén (vorticity) waves, and also breaks spatial statistical isotropy. A homogeneous magnetic field hence induces non-zero off-diagonal correlations between temperature multipole coefficients. In particular, non-zero correlations between l and $l \pm 2$ multipole coefficients are given by (Durrer et al. 1998)

$$D_l(m) = \langle a_{l-1,m}^* a_{l+1,m} \rangle = \langle a_{l+1,m}^* a_{l-1,m} \rangle. \quad (1)$$

Here the power spectrum $D_l(m)$ depends on the primordial vorticity perturbation spectrum (which we assume to be given by a simple power law $P_\Omega(k) \propto k^n/k_D^{n+3}$), and the Alfvén speed v_A . The presence of a non-zero $D_l(m)$ has a simple physical explanation: the temperature anisotropy correlation between two points on the sky depends not only on the angular separation between the two points but also on their orientation with respect to the magnetic field.

An observational test to detect (or constrain) the presence of a homogeneous cosmological magnetic field is based on computing the D_l spectrum of CMB anisotropy data. Chen et al. (2004) use the WMAP data to constrain the magnetic field amplitude (at illustrative value of vorticity spectral index $n = -7$ and $n = -5$) to be less than about $10^{-8} - 10^{-9}$ Gauss at three standard deviation.

A more realistic case¹ to consider is a stochastic magnetic field with a (Gaussian random) two-point correlation spectrum (Pogosian et al. 2002):

$$\langle B_i^*(\mathbf{k}) B_j(\mathbf{k}') \rangle = (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}') [P_{ij} P_B(k) + i \epsilon_{ijl} \hat{k}_l P_H(k)], \quad (2)$$

where $P_B(k)$ ($\propto \langle |\mathbf{B}|^2 \rangle$) and $P_H(k)$ ($\propto \langle \mathbf{B} \cdot (\nabla \times \mathbf{B}) \rangle$) are the symmetric and helical magnetic field power spectra, respectively (we assume that both are given by simple power laws), the plane projector $P_{ij} \equiv \delta_{ij} - \hat{k}_i \hat{k}_j$, ϵ_{ijl} is the totally antisymmetric tensor, and $\hat{k}_i = k_i/k$. The possibility of generating helical magnetic fields is discussed in Vachaspati (2001) and Sigl (2002). The symmetric part of the magnetic field in eq.(2) contributes to the CMB temperature and polarization anisotropies via induced vector and tensor perturbations (for

¹For cosmological magnetic field generation mechanisms see, e.g., Turner and Widrow (1988), Carroll and Field (1991), Ratra (1992), Dolgov and Silk (1993), and Enqvist and Olesen (1994)

details see Mack et al. 2002)². $P_B(k)$ induces parity-even CMB fluctuations, with the following maximum rate of growth with respect of l

$$C_l^{\theta\theta(V)} \propto l^2, \quad C_l^{EE(V)} \propto l^2, \quad C_l^{BB(V)} \propto l^2 \quad C_l^{\theta E(V)} \propto l^2 \quad (3)$$

For a vector perturbation the BB -power spectrum is slightly larger than the EE one, whereas the EE and θE power spectra are approximately equal ³While $n_S \rightarrow -3$ corresponds to the symmetric magnetic field power spectrum being scale-invariant, the CMB vector power spectra are not flat for this value.

For tensor perturbations, the parity-even CMB power spectra generated from the symmetric magnetic field power spectrum are (Durrer et al. 2000, Mack et al. 2002):

$$C_l^{\theta\theta(T)} \propto l, \quad C_l^{EE(T)} \propto l, \quad C_l^{BB(T)} \propto l, \quad C_l^{\theta E(T)} \propto l \quad (4)$$

For magnetic field induced gravitational wave contribution to the CMB anisotropies, the E polarization power spectrum is slightly larger than the B one. For $n_S > -3/2$ the polarization power spectra are comparable to the temperature power spectra. This is due to the fact that both the temperature and polarization fluctuations are dominated by the intrinsic temperature quadrupole moments, which arise from the gravitational wave solution \dot{h} instead of being induced via free streaming dipoles as in the case of a vector perturbations. Also, for $n_S > -3/2$ the magnetic source term for the tensor mode is approximately independent of k and the resulting power spectra have the well known behaviour for a white noise source: $C_l l^2 \propto l^3$. As expected, for a scale-invariant magnetic field with $n_S \rightarrow -3$, the tensor part of the CMB power spectra is flat. Note that our analytical approximations are valid for $l < 500$ for the vector mode and for $l < 100$ for the tensor mode (due to the damping of gravitational waves when they enter horizon at decoupling), see Caprini et al. (2004) for detailed discussion. Comparison with the WMAP CMB data (Bennett et. al. 2003) constrains the magnetic field amplitude to be less than about 10^{-9} Gauss ⁴.

²CMB temperature and polarization anisotropy vector mode contributions for a magnetic field spectrum peaked at a fixed value of k are given in Subramanian and Barrow (1998), Seshadri and Subramanian (2001), and Subramanian et al. (2003). CMB temperature anisotropy induced by gravitational waves generated by a magnetic field are discussed in Durrer et al. (2000)

³Temperature anisotropies are dominated by the vector dipole term, which correlates poorly with the radial function corresponding to E polarization (Hu and White 1997), so the θE power spectrum is dominated by a subdominant temperature contribution arising from the vector quadrupole term, which then coincidentally renders the spectrum in a form approximately identical to the E polarization power spectrum itself.

⁴Constraints of a similar magnitude result from considering resonant photon-graviton conversion (Chen 1995), and from the distortion of the CMB (Jedamzik et al. 2000)

A magnetic field with non-zero helicity ($P_H(k)$) will induce additional effects (Pogosian et al. 2002). In particular, the presence of a helical part results in non-zero parity-odd CMB power spectra, such as C_l^{EB} and $C_l^{\theta B}$. Also, a helical magnetic field will generate gravitational waves with parity odd spectra (Caprini et al. 2004). Using the linear polarization basis, $e_{ij}^T = (\mathbf{e}_1 \times \mathbf{e}_1 - \mathbf{e}_2 \times \mathbf{e}_2)_{ij}$, $e_{ij}^\times = (\mathbf{e}_1 \times \mathbf{e}_2 + \mathbf{e}_2 \times \mathbf{e}_1)_{ij}$ the helical part of the magnetic tensor source $g(k)$ can be directly connected with gravitational waves h^T and h^\times ($h_{ij} = e_{ij}^T h^T + e_{ij}^\times h^\times$) correlations:

$$\langle h^{\star T}(\mathbf{k}) h^\times(\mathbf{k}') - h^{\star T}(\mathbf{k}) h^\times(\mathbf{k}') \rangle \propto i \delta(\mathbf{k} - \mathbf{k}') g(k) \quad (5)$$

A magnetic field with helicity will also induce non-decaying vorticity waves (Pogosian et al. 2002). Both modes (vector and tensor) generate CMB temperature and polarization anisotropies. The helical part contributions to parity-even total CMB power spectra are negative, but due to the causality restriction, $P_B(k) > |P_S(k)|$ and $n_S \leq n_A$, the total C_l 's are positive. The ratio between the helical and symmetric part contributions to the parity-even CMB power spectra $C_{l,H}/C_{l,S}$ depends on the corresponding indices n_H and n_S , as well as on $P_H(k)$ and $P_B(k)$. The parity-odd power spectra are generated by $P_H(k)$, but are dependent on both the spectral amplitude and index. (For the tensor mode see Caprini et al. (2004) and for the vector mode a paper is in preparation).

As mentioned above, the presence of a cosmological magnetic field results also in the rotation of the CMB polarization plane via the Faraday effect (Kosowsky and Loeb 1996). Assuming that the rotational effect on polarization generated by the magnetic field itself is a second order effect, and also that only scalar perturbations are present, Faraday rotation will generates B-polarization. In a current project (Kosowsky et al. 2004) we study the Faraday rotation effect (and resulting B-polarization signal) due to a stochastic magnetic field. We show that an average rotation measure is independent of the helical part of magnetic field. Hence a measurement of the rotation measure can constrain the symmetric part of the magnetic field spectrum. The resulting B polarization depends on the initial polarization spectrum ($C_{l,in}^{EE}$) and on the rotation angle power spectrum ($C_l^{\alpha\alpha}$) as (Kosowsky et al. 2004):

$$\langle a_{l'm'}^{B*} a_{lm}^B \rangle = \delta_{ll'} \delta_{mm'} N_l^2 \sum_{l_1 l_2} N_{l_2}^2 K(l, l_1, l_2)^2 C_{l_2,in}^{EE} C_{l_1}^{\alpha\alpha} \frac{(2l_1 + 1)(2l_2 + 1)}{4\pi(2l + 1)} \quad (6)$$

$$(C_{l_1 0 l_2 0}^{l_0})^2$$

where

$$K(l, l_1, l_2) \equiv -\frac{1}{2} (L^2 + L_1^2 + L_2^2 - 2L_1 L_2 - 2L_1 L + 2L_1 - 2L_2 - 2L), \quad (7)$$

with $L = l(l + 1)$, $N_l \equiv (2(l - 2)!/(l + 2)!)^{1/2}$, and $C_{l_1 0 l_2 0}^{l_0}$ are Clebsh-Gordon coefficients.

Assuming precise measurements of CMB temperature and polarization anisotropies, Faraday rotation allow us to reconstruct the symmetric magnetic field spectrum, and since

the total CMB power spectra depend on both the symmetric and the helical parts of the magnetic field spectrum, we can also constrain magnetic helicity. Also, there is the theoretical possibility of reconstructing magnetic helicity from the magnetic-field-generated gravitational wave spectrum.

Our conclusions are as follows:

A homogeneous magnetic field, via generated Alfvén waves, induces non-zero off-diagonal correlations of CMB anisotropies multipoles coefficients. The magnetic field can thus be constrained by testing CMB data for non-gaussianity.

A helical magnetic field generates gravitational waves with parity odd spectra. This could serve, in principle, as a method for the detection of the helicity of the magnetic field.

The Faraday rotation measurement can’t constrain magnetic helicity. Thus, only the symmetric part of magnetic field spectrum can be reconstructed from RM measurements.

Current CMB data constrains the amplitude of a cosmological magnetic field to be less than order 10^{-9} Gauss.

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